CHAPTER 1 Motion

Motion is one of the more common events in your surroundings. You can see motion in natural events such as clouds moving, rain and snow falling, and streams of water moving, etc. Motion can also be seen in the activities of people who walk, jog or drive various vehicles from place to place. Motion is so common that you may think that everyone understand the concepts of motion. But history indicates that it was only during the past three hundred years or so that people began to understand motion correctly. The foundations for the study of motion were laid down more than 300 years ago by Galileo in Italy and later by Isaac Newton in England.
The study of motion comes in the branch of physics called '**mechanics**'. It is broken down into two parts, kinematics and dynamics. **Kinematics** is the "how" of motion, that is, the study of how objects move, without concerning that why they move. **Dynamics** is the "why" of motion. In dynamics, we are concerned with the causes of motion, which is the study of forces.

1.1 Position and reference point

Suppose you live in near Allen career institute (Samanvaya building), Kota. Fig.1 shows the aerial view (top view) of your locality. To reach Allen's Samanvaya building, you follow a path shown in the fig.1. Your house is the starting place for you to find the location, or position, of Allen's Samanvaya building.

- A **reference point** is a starting point used to describe the position of an object. A reference point is also called the **origin**.

To describe an object's position, three things must be included in the description : (i) a reference point, (ii) a direction from the reference point, (iii) distance from the reference point (the length of the line segment joining the reference point and the object).

For example, in the fig.1, choose your house as the reference point. Next, choose a direction from the reference point, let it be 'toward the ABC school' (see fig.1). Finally, give the distance from the reference point; let it be 1.2 km.

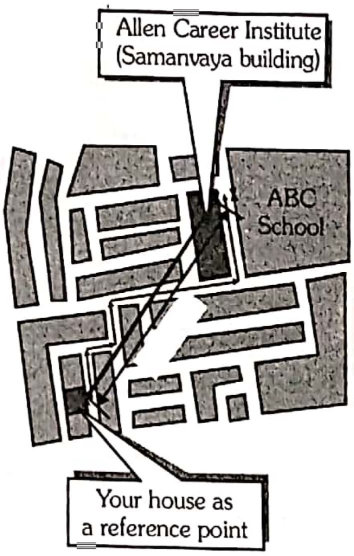


Fig.1 A reference point is needed in order to describe the location of an object.

How to describe the reference direction ?

One way of indicating the direction is to use a plus (+) or a minus (–) sign. The plus sign can be the direction from the reference point is in the reference direction (see fig.2). A minus sign means the direction is opposite to the reference direction. For example, plus (+) sign can be used to indicate 'toward the school' in the fig.1 and minus (–) sign to indicate 'away from the school'.

- The position of an object can be described as a distance from the origin together with a plus or minus sign that indicates the direction.
- Cartesian system of representing position of a particle is shown in fig.3

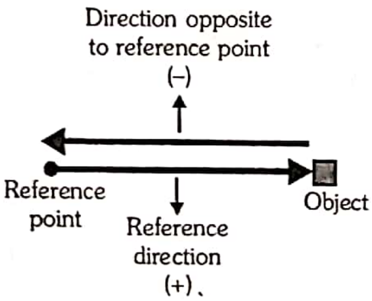


Fig.2 Describing the reference direction : sign conventions.

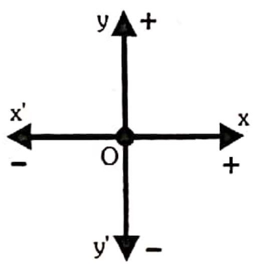


Fig.3 Cartesian system : Sign convention for position

NUMERICAL CHALLENGE 1.1

In the fig.4, a car is parked at 100 m from the centre of a crossing. Also a tree is located 75 m from the car as shown in fig.4. The reference direction and its sign is mentioned in the fig.4. How will you express the car's position if

(i) when the centre of the crossing is taken as reference point (or origin),

(ii) when the tree is taken as reference point (or origin).

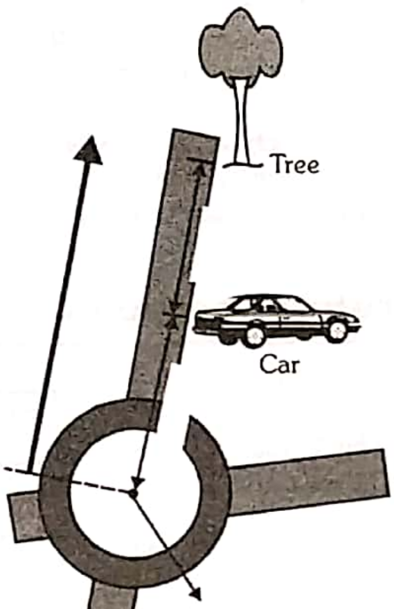


Fig.4

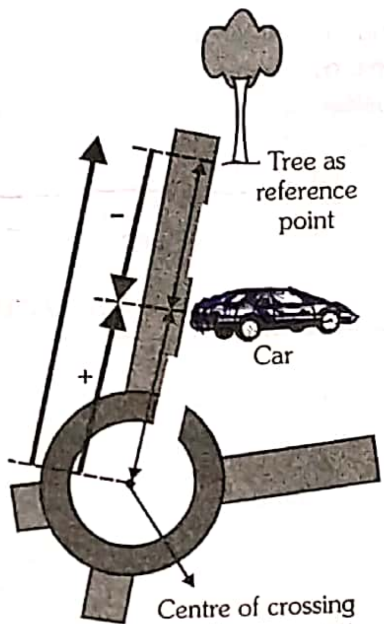


Fig.5

Solution

(i) Here, the reference point is centre of the crossing. The car is 100 m away from this reference point. If draw a direction from this reference point to the car, this direction is same as the reference direction (see fig.4). Thus, the sign of this direction must be taken positive. Hence, the position of car is expressed or represented as **+ 100 m**.

(ii) Here, the reference point is the tree. The car is 75 m away from this reference point. If draw a direction from this reference point to the car, this direction is opposite to the reference direction (see fig.5). Thus, the sign of this direction must be taken negative. Hence, the position of car is expressed or represented as **– 75 m**.

Thus, an object's position is its location compared to other things. Position of an object is not absolute, it is a variable that gives location of an object **relative** to a reference point or origin.

1.2 Understanding motion

Consider a ball that you notice one morning in the middle of a lawn. Later in the afternoon, you notice that the ball is at the edge of the lawn, against a fence and you wonder if the wind or some person moved the ball. You do not know if the wind blew it at a steady rate, or even if some children kicked it all over the yard. All you know for sure is that the ball has been moved because it is in a different position after some time passed. These are the two important aspects of motion :

(1) A change of position (2) The passage of time

Moving involves a change of position during some time period. Motion is the act or process of something changing position. The motion of an object is usually described with respect to a stationary object. Such a stationary object is said to be 'at rest'.

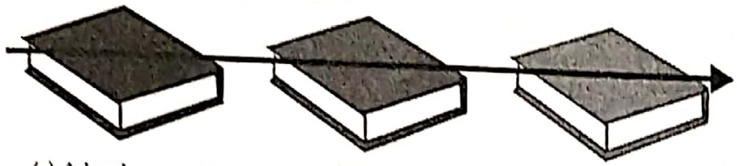
- **Motion** is a change in an object's position compared to a fixed object. If you ride in a car, your position changes compared to a tree or an electric pole.
  - An object is said to be at **rest** if it does not change its position with time.

Rest and motion are relative terms

Imagine that you are traveling in an automobile with another person. You know that you are moving across the land outside the car since your location on the highway changes from one moment to another. Observing your fellow passenger shows that there is no change of position. You are in motion relative to the ground but you are not in motion relative to your fellow passenger. The motion of any object or body is the process of a change in position 'relative' to some reference object or location.

Translational motion (or translatory motion)

Motion of a body in which all the points in the body follow parallel paths is called 'translational motion'. It is a motion in which the orientation of an object remains the same throughout the journey. The path of a translatory motion can be straight or curved (see fig.6 & fig.7).



(a) A book moved along a straight path without changing its orientation

(b) A book moved along a curved path without changing its orientation

Fig.6 Translational motion

On the basis of the path travelled by an object, the translational motion can classified as :

(1) **Rectilinear motion :** If an object moves in a straight line, its motion is called rectilinear motion or one dimensional motion. Motion of car along a straight path, motion of a piston in the cylinder are examples of rectilinear motion.

(2) **Curvilinear motion :** If an object moves along a curved path without change in its orientation, its motion is called curvilinear motion. Motion of a car along a curved or circular path, motion of an athlete on a circular track are examples of curvilinear motion.

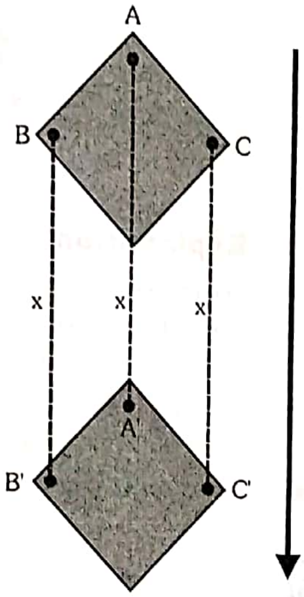


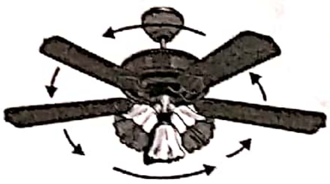
Fig.7 Translational motion : The particles of the object shown cover same distance in a given time.

Rotational motion (Rotatory motion)

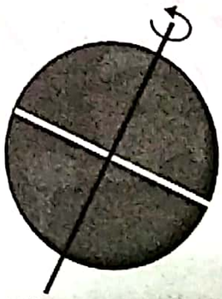
Motion of a body turning about an axis is called rotational motion. In other words, 'a motion in which an object spins about a fixed axis is called rotational motion'. It is a motion in which the orientation of an object continuously changes throughout the motion. The path of an object in a rotational motion is always circular.

Some examples of rotational motion are :

(1) The Earth's spin on its axis.
(2) Motion of a fan or motor.
(3) Motion of blades of windmill.
(4) Motion of a spinning top.
(5) Motion of a grinding stone.



(a) Motion of a ceiling fan



(b) Motion of Earth about its axis

(c) Motion of a spinning top

Fig.8 Some examples of rotational motion

- In rotational motion, the particles of the object move through the unequal distances in a given time depending on their location in the object (see fig.9). The particle which is located near the axis of rotation, covers less distance as compared to the particle that is located far away from the axis.

  - In translational motion at any instant of time, every particle of the body has the same velocity while in rotational motion at any instant of time particles of the body have different velocities depending on their position from the axis of rotation.
  - In rotation of a body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

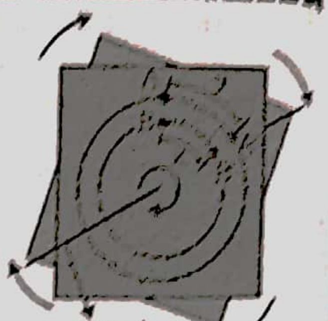


Fig.9 Rotational motion : Particles cover unequal distances in a given time.

CONCEPTUAL CHALLENGE 1.1

In fig.10, motion of a frying pan used in kitchen is shown. Is the motion of the frying pan a translational motion ? Can it be considered as rotational motion ? Explain.

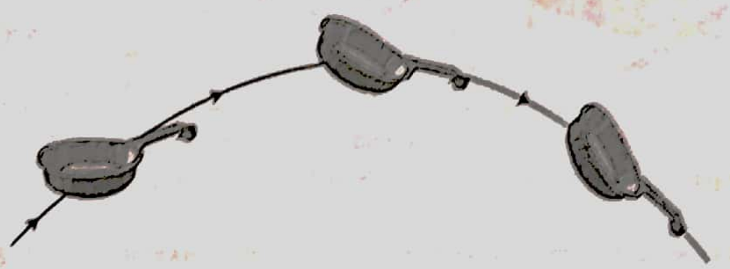


Fig.10 Conceptual challenge 1.1

Explanation

The motion of frying pan shown in fig.10 cannot be considered as translational motion though it is moving along a curved path. This is because its orientation is changing during its journey. Also, the motion of frying pan cannot be considered as rotational motion though it is spinning. This is because, rotation means spinning of an object about a fixed axis. Here, the flask is not spinning about a fixed axis. This type of motion is 'a combination of translational motion and rotational motion'.

- Motion of a car or cycle wheels is a combination of translational and rotational motion (see fig.11).

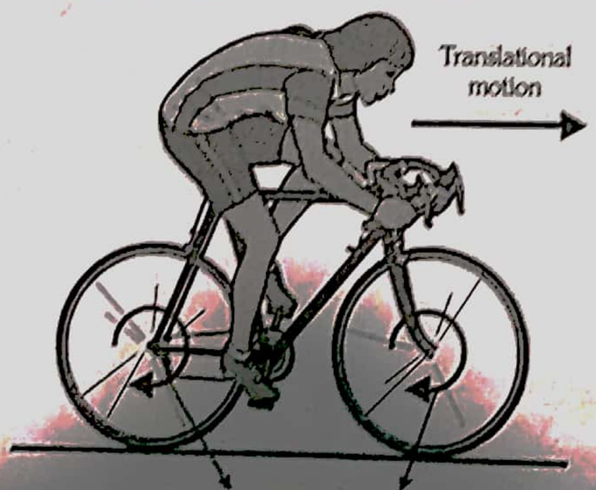


Fig.11 Motion of a cycle wheel is a combination of translational and rotational motion. Motion of a wheel is also called 'rolling motion'.

1.3 Uniform and non-uniform motion

Uniform motion

If a body covers equal distances in equal intervals of time in a particular direction, its motion is called 'uniform motion'. In other words, 'if the velocity of a body is constant, its motion is called uniform motion'.

- A uniform motion always takes place in straight line. Any motion along a curved path is not a uniform motion.

Examples : (i) A car moving with a constant speed in straight line (ii) Motion of an athlete along straight path with constant speed.

- In real world, we rarely see uniform motion. A body can be in uniform motion for a short time period then we have to take turn, reduce speed or increase speed according to our need.

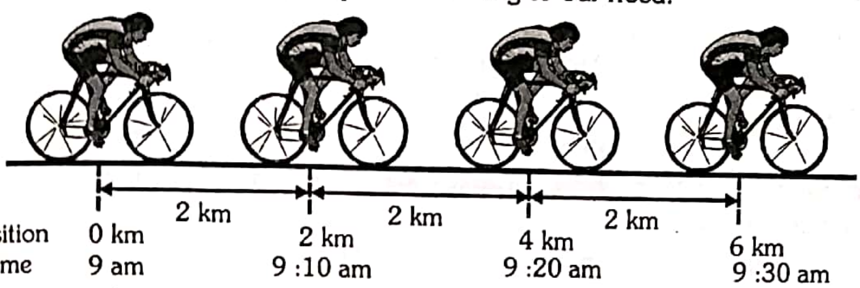


Fig.12 Motion of this cyclist is uniform motion ; she is covering equal distances in equal intervals of time; she is moving in straight line in a particular direction.

Non-uniform motion

If velocity of a body is variable, its motion is called non-uniform motion.

- For non-uniform motion,

(a) Magnitude of velocity is variable, or

(b) Direction of velocity is variable, or

(c) Both the magnitude as well as direction of the velocity is variable.

- If a body covers unequal distances in equal intervals of time, its motion is called 'non-uniform motion'.
- Even if a body covers equal distances in equal intervals of time but it changes its direction, still its motion is said to be 'non-uniform'.
- Motion of a particle along a curved path is always a non-uniform motion. If a particle changes its direction during the journey, its motion is always non-uniform.

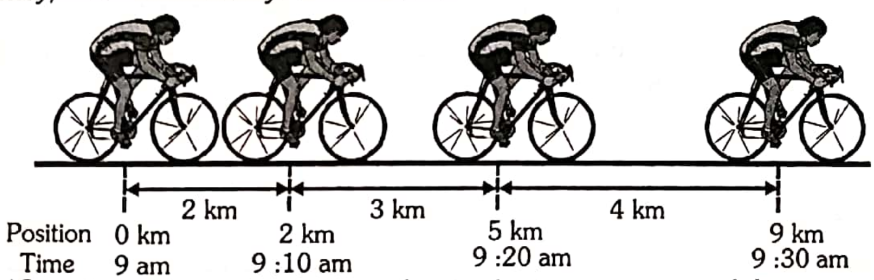


Fig.13 Motion of this cyclist is 'non-uniform' ; she is covering unequal distances in equal intervals of time; she is moving in straight line in a particular direction.

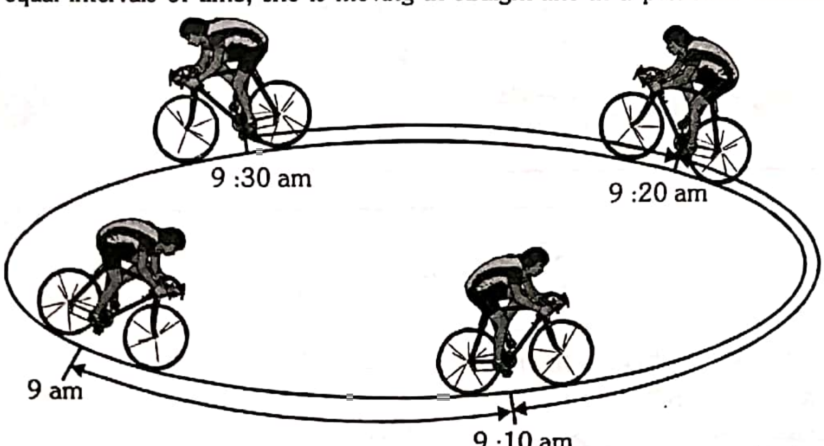


Fig.14 This cyclist is moving along a circular path ; she is covering equal distances in equal intervals of time ; still this is a non-uniform motion as this is not in a straight line.

Distance and displacement

Distance

The length of the actual path between initial and final positions of a moving object is called 'distance'.

- Distance is a scalar quantity. • Distance depends on the path. • Distance is always taken positive.

Unit of distance : In S.I. system unit of distance is metre (m). Some other popular units are millimetre (mm), centimetre (cm), kilometre (km).

Displacement

The shortest distance between the initial position and the final position of the particle is called displacement.

It is also defined as the change in the position of the particle.

Displacement = $x_f - x_i$

Where, x_f = final position ; x_i = initial position.

- Displacement is a vector quantity, its direction is always taken from initial position to final position.
- Displacement depends only on initial position and final position, does not depend on path.
- Displacement of a particle in motion can be positive, negative or even zero.

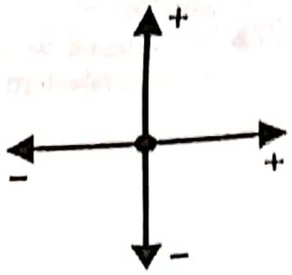


Fig.15 Sign convention for displacement.

Unit of displacement : Units of distance and displacement are same as both represent some length. Thus, in S.I. system unit of displacement is metre (m). Some other popular unts are millimetre (mm), centimetre (cm), kilometre (km).

Let us understand the distance and displacement using some real life situation. Suppose a boy walks in a park, as shown in fig.16. His initial position is A. He first walks a distance of 30 m due east. Then, he walks 40 m due north. Here, the distance travelled by him is AB + BC = 30 m + 40 m = **70 m.**

His displacement is given by,

$$BC = \sqrt{AB^2 + BC^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = \mathbf{50\ m}$$

- Distance is always greater than or equal to the magnitude of displacement.
  - Whenever a particle changes its direction or follows a curved path, distance is always greater than the magnitude of displacement.
  - Distance is exactly equal to displacement (i) when it follows a straight path without changing its direction (ii) when its is in uniform motion.

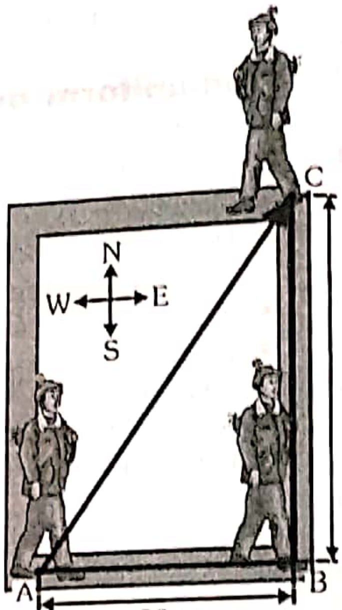


Fig.16 Understanding distance and displacement

NUMERICAL CHALLENGE 1.2

In the fig.17, a car moves on the road from the 20 km mark (its initial position) to the 100 km mark. After that, it reverses and moves back to the 50 km mark (its final position). Find the displacement and distance travelled by the car.

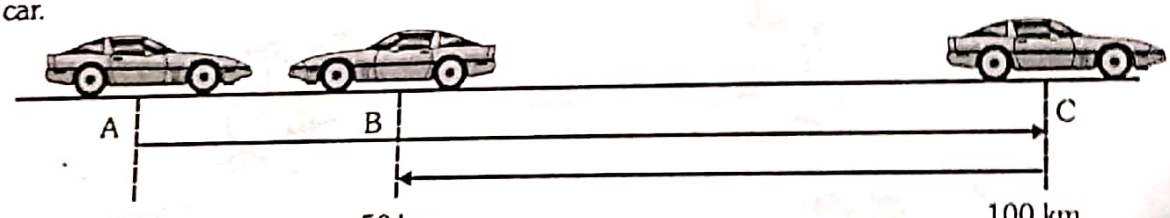


Fig.17 Numerical challenge 1.2

Solution

Given, initial position, x_i = + 20 km ; final position, x_f = +50 km

Displacement = $x_f - x_i$ = (+50) – (+20) = **+ 30 km**

Now, distance travelled by car from A to C, AC = 100 – 20 = 80 km

Distance travelled by car from C to B, BC = 100 – 50 = 50 km

Total distance travelled by car = AB +BC = 80 + 50 = **130 km**

1.5 Speed and velocity

Speed

The distance travelled by a particle per unit time is called speed.

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{time}}}$$

- Speed is a scalar quantity.
- Speed depends on the path.
- $\boxed{1\ \text{km/h} = \frac{5}{18}\ \text{m/s}}$
- Speed gives no idea about the direction of motion of the object.
- Speed can never be negative ; in motion, it is taken positive ; at rest, it is zero.

Unit of speed : S.I. system - metre/second (m/s); C.G.S.system - centimetre/second (cm/s).

Uniform speed : An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time. That is, magnitude of speed is constant.

Non uniform speed : An object is said to be moving with a variable speed if it covers unequal distances in equal intervals of time. That is, magnitude of speed is variable.

Average Speed : When an object is moving with a variable speed, then the average speed of the object is thought to be that constant speed with which the object covers the same distance in a given time interval as it does while moving with variable speed during the same time interval.

Average speed is the ratio of the total distance travelled by the object to the total time taken.

$$\boxed{\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$

Instantaneous speed : The speed of the body at any instant of time is called instantaneous speed.

- **Speedometer** of the vehicle measures its instantaneous speed.
- In uniform motion of a particle, the instantaneous speed is equal to its average speed.

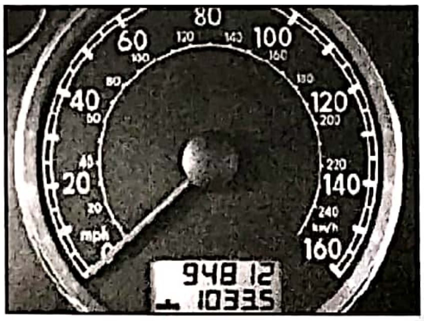


Fig.18 A speedometer measures speed but not velocity.

Velocity

The rate of change of displacement is called velocity.

- Velocity is a vector quantity.
- Velocity can be negative, positive or zero.
- The direction of average velocity is same as that of the total displacement.
- If average velocity for a journey is positive, it may have a negative instantaneous velocity at some point of time during the journey and vice-versa.

Unit of velocity : S.I. system - metre/second (m/s); C.G.S.system - centimetre/second (cm/s).

Instantaneous velocity : It is the velocity at some particular instant of time.

Average velocity : It is the ratio of total displacement to the total time taken.

$$\boxed{\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}}$$

Uniform Velocity : A particle is said to have uniform velocity, if the magnitude as well as the direction of its velocity remains constant. It is possible only when the particles moves in straight line without changing its direction.

Non-uniform Velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of its velocity changes (or both changes).

- In uniform motion of a particle, the instantaneous velocity is equal to its average velocity.

- Average speed is always greater than or equal to the magnitude of average velocity.
  - Whenever a particle changes its direction or follows a curved path, average speed is always greater than the magnitude of average velocity.
  - Average speed is exactly equal to average velocity when it follows a straight path without changing its direction.

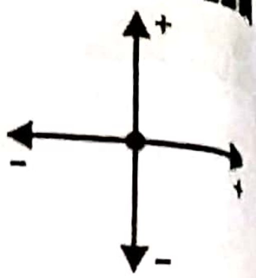


Fig.19 Sign convention for velocity

- If body covers distances x_1, x_2, x_3,---- with speeds v_1, v_2, v_3,---- respectively in same direction then average speed/average velocity of body is given by,

$$\boxed{v_{average} = \frac{x_1 + x_2 + x_3 + ---}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \frac{x_3}{v_3} + ---}}$$

  - **Case of half journey :** If body covers equal distances $x_1 = x_2 = x$ (let), with different speeds i.e, v_1 and v_2

$$v_{average} = \frac{x + x}{\left(\frac{x}{v_1} + \frac{x}{v_2}\right)} = \frac{2x}{x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2}{\left(\frac{v_2 + v_1}{v_1 v_2}\right)} = \boxed{\frac{2v_1 v_2}{v_1 + v_2}}$$

  - If a body covers three equal distances with speeds v_1, v_2 and v_3 respectively, then average speed is given by,

$$v_{average} = \frac{x + x + x}{\left(\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}\right)} = \frac{3x}{x\left(\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}\right)} = \frac{3}{\left(\frac{v_1 v_2 + v_2 v_3 + v_3 v_1}{v_1 v_2 v_3}\right)} = \boxed{\frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}}$$

- If a body travels with speeds v_1, v_2, v_3 ,---- during time intervals t_1, t_2, t_3, --- respectively then the average speed of the body is given by,

$$\boxed{v_{average} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + ---}{t_1 + t_2 + t_3 + ---}}$$

  - If the two given time intervals are same i.e., $t_1 = t_2 = t$ (let), then,

$$v_{average} = \frac{v_1 t + v_2 t}{t + t} = \frac{(v_1 + v_2)t}{2t} = \boxed{\frac{v_1 + v_2}{2}}$$

  - If the three given time intervals are same i.e., $t_1 = t_2 = t_3 = t$ (let), then,

$$v_{average} = \frac{v_1 t + v_2 t + v_3 t}{t + t + t} = \frac{(v_1 + v_2 + v_3)t}{3t} = \boxed{\frac{v_1 + v_2 + v_3}{3}}$$

NUMERICAL CHALLENGE 1.3

An auto travels at a rate of 25 km/hr for 4 min. then 50 km/hr for 8 min., finally at 20 km/hr for 2 min., find the distance travelled in km and the average speed for complete trip in m/s.

Solution

Given, $v_1 = 25$ km/hr ; $v_2 = 50$ km/hr ; $v_3 = 20$ km/hr ;

$t_1 = 4$ min $= (4/60)$ hr ; $t_2 = 8$ min $= (8/60)$ hr ; $t_1 = 2$ min $= (2/60)$ hr

Distance travelled, $s = v_1 t_1 + v_2 t_2 + v_3 t_3$

$$= 25 \times \frac{4}{60} + 50 \times \frac{8}{60} + 20 \times \frac{2}{60} = \frac{100 + 400 + 40}{60} = \frac{540}{60} = \mathbf{9\ km}$$

Total time, $t = t_1 + t_2 + t_3 = \frac{4}{60} + \frac{8}{60} + \frac{2}{60} = \frac{14}{60} = \frac{7}{30}$ hr

Average speed, $v = \frac{s}{t} = \frac{9\ km}{(7/30) hr} = \frac{9 \times 30}{7}$ km/hr $= \frac{9 \times 30}{7} \times \frac{5}{18}$ m/s $= \frac{75}{7}$ m/s $= \mathbf{10.7\ m/s}$

NUMERICAL CHALLENGE 1.4

On a 60 km track, a train travels the first 30 km at a uniform speed of 30 km/hr. How fast must the train travel the next 30 km so as to average 40 km/hr for entire trip ?

Solution

Given, speed for first 30 km, $v_1 = 30$ km/hr ; speed for next 30 km, $v_2 = ?$;
average speed, $v_{average} = 40$ km/hr.
This is a case of half journey, therefore, we can apply the formula for half journey directly.

$$v_{average} = \frac{2v_1v_2}{v_1+v_2} \quad \text{or} \quad 40 = \frac{2(30)v_2}{30+v_2} \quad \text{or} \quad 40 = \frac{60v_2}{30+v_2} \quad \text{or} \quad 2 = \frac{3v_2}{30+v_2}$$

or $2(30 + v_2) = 3v_2$ or $60 + 2v_2 = 3v_2$
or $v_2 =$ **60 km/hr**

1.6 Acceleration

- The rate of change of velocity is called acceleration.
  - It is a vector quantity. Its direction is same as that of change in velocity and NOT of the velocity.
  - It is NOT the rate of change of speed. For example, when a body moving with constant speed along a circular path, there is no change in its speed but there is a change in velocity as its direction is changing continuously at every point. Thus, there must be some acceleration of the body.
  - A change in velocity occurs when (i) only its direction changes, e.g. uniform circular motion. (ii) only its magnitude changes. e.g. a ball dropped from a certain height under gravity (iii) both magnitude as well as direction changes, e.g. a projectile motion. In all these cases, there MUST be some acceleration present in the motion.
  - Whenever velocity and acceleration are in same direction, the velocity of a particle increases. Such motion is called an accelerated motion. Such an acceleration for numericals is usually taken as 'positive acceleration'.

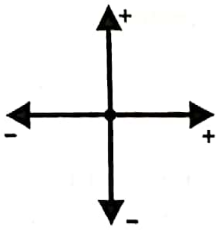


Fig.20 Sign convention for acceleration

  - Whenever velocity and acceleration are in opposite direction, the velocity of a particle decreases. Such motion is called retarded motion. Such an acceleration for numericals is usually taken 'negative acceleration' and also called 'retardation' or 'deceleration'.

Acceleration, $\boxed{a = \frac{v-u}{t}}$

Unit of acceleration : S.I. system - metre/(second)^2 (m/s^2); C.G.S.system - centimetre/(second)^2 (cm/s^2).

Non-uniform motion with constant acceleration (uniformly accelerated motion)

It is a motion in which acceleration is constant in both magnitude as well as direction.

- It is a non-uniform motion.

Equations of motion for a uniformly accelerated motion are :

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 = u^2 + 2as$ (iv) $s = \left(\frac{v+u}{2}\right)t$ (v) $v_{average} = \frac{v+u}{2}$

Where, u = initial velocity ; v = final velocity ; s = distance travelled ; t = time taken, a = acceleration.

- Distance travelled in nth second (i.e., in a particular second) is given by,

$$\boxed{s_{nth} = u + \frac{1}{2}a(2n-1)}$$

NUMERICAL CHALLENGE 1.5

A body travels 200 cm in first two seconds and 220 cm in next four seconds. What will be the velocity at the end of the seventh second.

Solution

Let u be the initial velocity, a be the acceleration of the body.

For first two seconds, distance travelled is 200 cm i.e., for t = 2 ; s = 200 cm.

Using second equation of motion, $s = ut + \frac{1}{2}at^2$, we get,

$$200 = u(2) + \frac{1}{2}a(2)^2 \quad \text{or} \quad 200 = 2u + 2a$$

or $u + a = 100$ ----- (1)

For next four seconds, distance travelled is 220 cm. This means for first (2 + 4) second i.e., first 6 seconds, the distance travelled is 200 + 220 = 420 cm. Here, at t = 6 s ; s = 420 cm. Again using second equation of motion, we get,

$$420 = u(6) + \frac{1}{2}a(6)^2 \quad \text{or} \quad 420 = 6u + 18a$$

or $u + 3a = 70$ ----- (2)

Subtracting eq.(1) from eq.(2), we get,

$$\begin{array}{rl} & u + 3a = 70 \\ - & u + a = 100 \\ \hline & 2a = -30 \end{array}$$

or $a = -15 \text{ cm/s}^2$

Putting the value of a in eq.(1), we get, $u - 15 = 100$ or $u = 115$ cm/s

Now, we have to find velocity at the end of seventh second. Using first equation of motion, $v = u + at$ we get,

$v = 115 + (-15)(7) = 115 - 105 =$ **10 cm/s**

NUMERICAL CHALLENGE 1.6

A particle moving with constant acceleration form A to B in straight line AB has velocities 'u' and 'v' at A and B respectively. Find the velocity at C, the mid point of AB.

Solution

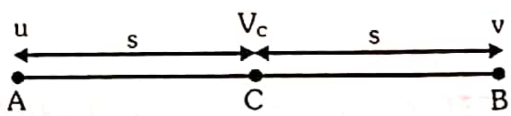


Fig.21 Numerical challenge 1.6

Since C is the mid point of AB,

AC = CB = s (let)

Velocity at A, $V_A = u$; Velocity at B, $V_B = v$;

Velocity at C, $V_C = ?$

Applying third equation of motion between A and C, we get,

$V_C^2 = V_A^2 + 2as$ or $V_C^2 = u^2 + 2as$ ----- (1)

Applying third equation of motion between C and B, we get,

$V_B^2 = V_C^2 + 2as$ or $v^2 = V_C^2 + 2as$ or $V_C^2 = v^2 - 2as$ ----- (2)

Adding eq.(1) + eq.(2), we get,

$V_C^2 + V_C^2 = (u^2 + 2as) + (v^2 - 2as)$

$$2V_C^2 = v^2 + u^2 \quad \text{or} \quad V_C^2 = \frac{v^2 + u^2}{2}$$

$$\text{or} \quad V_C = \sqrt{\frac{v^2 + u^2}{2}}$$

NUMERICAL CHALLENGE 1.7

A particle moving with uniform acceleration in a straight line covers 3 m in the 8th second and 5 m in the 16th second of its motion. Find the distance travelled by it from the beginning of the 6th second to the end of the 15th second.

Solution

Let u be the initial velocity, a be the acceleration of the particle.

Distance covered by the particle in 8th second is 3 m. Using the equation for s_{nth},

$3 = u + \frac{1}{2}a(2 \times 8 - 1)$ or $3 = u + \frac{1}{2}a(15)$ or $2u + 15a = 6$ ---- (1)

Distance covered by the particle in 16th second is 5 m. Again, using the equation for s_{nth},

$5 = u + \frac{1}{2}a(2 \times 16 - 1)$ or $5 = u + \frac{1}{2}a(31)$ or $2u + 31a = 10$ ---- (2)

Eq.(2) – eq.(1) $\Rightarrow (2u + 31a) - (2u + 15a) = 10 - 6$

or $16a = 4$ or $a = (1/4)\ m/s^2$

Using eq.(1), we get, $2u + 15 \times \frac{1}{4} = 6$ or $2u = 6 - \frac{15}{4} = \frac{9}{4}$ or $u = (9/8)\ m/s$

Now, we have to find the distance covered by the particle from the beginning of the 6th second to the end of the 15th second. At the beginning of the 6th second, total time elapsed is 5 second. First, we will find the velocity at the end of 5th second using first equation of motion,

$v = u + at$ or $v = \frac{9}{8} + \left(\frac{1}{4}\right)(5) = \frac{9}{8} + \frac{5}{4} = \frac{19}{8}\ m/s$

Now time taken between the beginning of the 6th second to the end of the 15th second is actually 10 seconds (6th, 7th, 8th, 9th, 10th, 11th, 12th, 13th, 14th, 15th). [**Caution :** If you subtract 15 – 6, you will get 9 seconds while actual time elapsed is 10 seconds]

Now, using second equation of motion, $s = ut + \frac{1}{2}at^2$, we get,

$$s = \left(\frac{19}{8}\right)(10) + \frac{1}{2}\left(\frac{1}{4}\right)(10)^2 = \frac{190}{8} + \frac{100}{8} = \frac{290}{8} = 36.25\ m$$

1.7 Free fall (motion under gravity)

Till 1600 AD, the teachings of the Greek philosopher Aristotle (384–322 BC) had held that *heavier objects fall faster than lighter ones.* The Italian physicist Galileo Galilei gave the present day ideas of falling objects. Now, it is an established fact that, in the absence of air resistance, all objects dropped near the Earth's surface fall with the same constant acceleration under the influence of the Earth's gravity.

- Free fall is the motion of an object subject only to the influence of gravity. An object is in free fall as soon as it is dropped from rest, thrown downward or thrown upward.

  Acceleration due to gravity : The constant acceleration of a freely falling body is called the acceleration due to gravity.

  - The acceleration due to gravity is the acceleration of an object in free fall that results from the influence of Earth's gravity. Its magnitude is denoted with the letter 'g'. The value of 'g' on the surface of Earth is nearly $9.8\ m/s^2$. In C.G.S. system, $g = 980\ cm/s^2$; in F.P.S. system, $g = 32\ ft/s^2$.

- Earth's gravity always pulls downward, so the acceleration (g) of an object in free fall is always downward and constant in magnitude, regardless of whether the object is moving up, down or is at rest, and independent of its speed.

  - If the object is moving downward, the downward acceleration makes it speed up; if it is moving upward, the downward acceleration makes it slow down.

Equations of motion of freely falling body

There are two main assumptions in free fall :

(1) Acceleration due to gravity (g) is constant throughout the motion and it acts vertically downwards.

(2) Air resistance is negligible.

Case 1 : An object thrown vertically upward and it returns after some time (see fig.22).

Let us consider an object thrown vertically upward with an initial velocity u, the acceleration due to gravity g acting vertically downward on it. Let after a time interval t, it achieves an height h and final velocity v.

Initial velocity = + u ; acceleration, a = − g ; distance travelled, s = + h

From first equation of motion, we have, $v = u + at$

or $v = (+u) + (-g)t$ or $v = u - gt$ ----- (1)

From second equation of motion, we have, $s = ut + \frac{1}{2}at^2$

or $+h = (+u)t + \frac{1}{2}(-g)t^2$ or $h = ut - \frac{1}{2}gt^2$ ----- (2)

From third equation of motion, we have, $v^2 = u^2 + 2as$

or $v^2 = (+u)^2 + 2(-g)(+h)$ or $v^2 = u^2 - 2gh$ ----- (3)

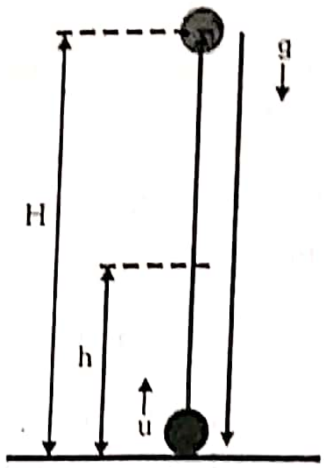


Fig.22

Time taken to reach maximum height :

At maximum height, v = 0

From eq.(1), we get, $0 = u - gt$ or $u = gt$ or $\boxed{t = \frac{u}{g}}$

Total time of journey :

Since g is constant throughout the motion, time taken to reach maximum height from the ground is equal to time taken to reach ground from the maximum height. That is, total time (T) of journey,

$$T = 2t = \frac{2u}{g} \quad \text{or} \quad \boxed{T = \frac{2u}{g}}$$

Maximum height achieved by the object :

Let the maximum height achieved be H. At maximum height, v = 0

From eq.(3), we get, $(0)^2 = u^2 - 2g(H)$ or $u^2 = 2gH$ or $\boxed{H = \frac{u^2}{2g}}$

- Here, the total distance covered, $s = 2H = 2\left(\frac{u^2}{2g}\right) = \frac{u^2}{g}$ while, the total displacement is zero.

Case 2 : An object is thrown vertically downward from a certain height H (see fig.23)

Let us consider an object thrown vertically downward with an initial velocity u, the acceleration due to gravity g is acting vertically downward on it. Let after a time interval t, it falls through a distance y and achieves a final velocity v.

Initial velocity = − u ; acceleration, a = − g ; distance travelled, s = − y ; final velocity = − v

From first equation of motion, we have, $v = u + at$

or $(-v) = (-u) + (-g)t$ or $-v = -u - gt$

or $-v = -(u + gt)$ or $v = u + gt$ ----- (1)

From second equation of motion, we have, $s = ut + \frac{1}{2}at^2$

or $-y = (-u)t + \frac{1}{2}(-g)t^2$ or $-y = -ut - \frac{1}{2}gt^2$

or $-y = -\left(ut + \frac{1}{2}gt^2\right)$ or $y = ut + \frac{1}{2}gt^2$ ----- (2)

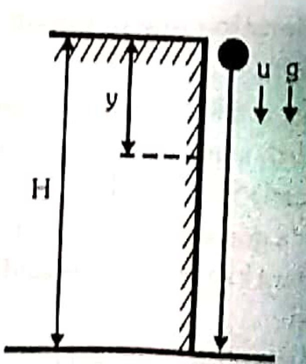


Fig.23

From third equation of motion, we have, $v^2 = u^2 + 2as$

or $(-v)^2 = (-u)^2 + 2(-g)(-y)$ or $\quad v^2 = u^2 + 2gy$ —— (3)

Velocity at ground : When the object reaches the ground, $y = -H$, then, from third equation of motion,

$v^2 = u^2 + 2as$ or $(-v)^2 = (-u)^2 + 2(-g)(-H)$

or $v^2 = u^2 + 2gH$ or $v = \sqrt{u^2 + 2gH}$

Time taken to reach the ground : When the object reaches the ground, $y = -H$, then, from second equation of motion,

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad -H = (-u)t - \frac{1}{2}gt^2$$

or $H = ut + \frac{1}{2}gt^2$. This is a quadratic equation that can be solved by factorisation or using quadratic formula.

- For numericals, we can assume acceleration due to gravity as + g for downward while – g for upward motion.

■ If an object is dropped from certain height, its initial velocity is taken zero i.e., $u = 0$. In such case the eqs.(1),(2),(3) will reduce to,

$$\boxed{v = gt \;;\; y = \frac{1}{2}gt^2 \;;\; v^2 = 2gy}$$

- **Velocity at ground :** When particle reaches the ground, $y = H$, then,

  $v^2 = 2gH$ or $v = \sqrt{2gH}$

- **Time taken to reach the ground :** When particle reaches the ground, $y = H$, then,

  $H = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2H}{g}}$.

Case 3 : An object thrown up from a certain height H or dropped from a rising balloon/helicopter :

Let us consider an object thrown vertically upward (see fig.24) from a certain height H with an initial velocity u, the acceleration due to gravity g is acting vertically downward on it. Also, if an object is dropped from a hot air balloon or a helicopter which is rising up into the atmosphere, the case will remain the same. This is because the initial velocity of a body dropped from a moving object is equal to the velocity of the moving object. In both cases, the object rises first, reaches a maximum height, then it moves downwards and finally reaches the ground.

Let after a time interval t, it moves a distance y and achieves a final velocity v.

Initial velocity = + u ; acceleration, a = – g ; distance travelled, s = y ; final velocity = v.

From first equation of motion, we have,

$v = u + at$

or $v = (+u) + (-g)t$

or $v = u - gt$ —— (1)

- In the eq.(1), if v comes positive, it means that object is moving upwards. If v comes negative, it means that object is moving downwards.

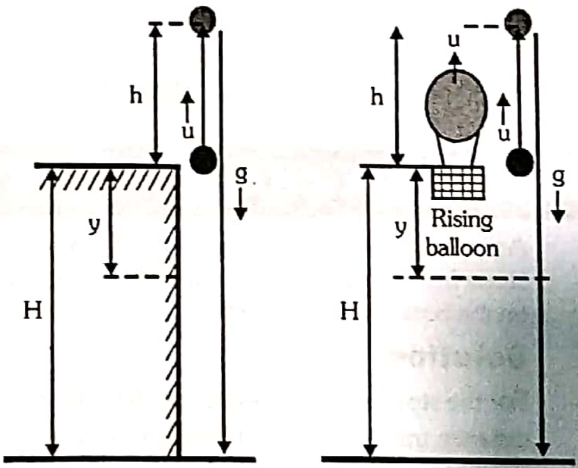


Fig.24

From second equation of motion, we have,

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad y = (+u)t + \frac{1}{2}(-g)t^2$$

or $\quad y = ut - \frac{1}{2}gt^2$ —— (2)

- In the eq.(2), if y comes positive, it means that object is above the initial point. If y comes negative, it means that object is below the initial point.

  From third equation of motion, we have, $v^2 = u^2 + 2as$

  or $(v)^2 = (+u)^2 + 2(-g)(y)$ or $\quad v^2 = u^2 - 2gy$ —— (3)

Velocity at ground : When particle reaches the ground, $y = -H$, then, from third equation of motion,

$v^2 = u^2 + 2as$ or $(-v)^2 = (-u)^2 + 2(-g)(-H)$

or $v^2 = u^2 + 2gH$ or $v = \sqrt{u^2 + 2gH}$

Time taken to reach the ground : When particle reaches the ground, y = – H, then, from second equation of motion,

$s = ut + \frac{1}{2}at^2$ or $-H = (+u)t - \frac{1}{2}gt^2$

or $H = -ut + \frac{1}{2}gt^2$. This is a quadratic equation that can be solved by factorisation or using quadratic formula.

- Let three balls 1, 2, and 3 are allowed to fall under gravity from the same height. Ball 1 is thrown vertically upward with speed u and it reaches the ground (see fig.25) in time t_1. Ball 2 is thrown vertically downward with the same speed u and it reaches the ground in time t_2. Ball 3 is dropped (i.e., u = 0) from the same height and it reaches ground in time t_3. Then, the relationship between t_1, t_2 and t_3 is given by,

$$\boxed{t_3 = \sqrt{t_1 t_2}}$$

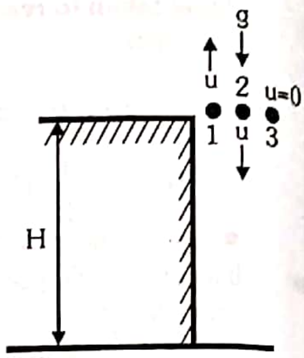


Fig.25

Case 4 : An object is dropped in a well and the sound of splash in water is heard after a certain time t.

Let us consider a well in which water level is present at a depth 'd' from the ground level. An object is dropped in it. When the object strikes the water surface, a splash (sound) is produced which reaches our ear after a very short time period (see fig.26).

Downward motion of object : It is a case of free fall i.e., motion under gravity. Initial velocity, u = 0 ; distance travelled, s = depth of well = d ; time taken, $t = t_1$

Now, from second equation of motion, $s = ut + \frac{1}{2}gt^2$

or $d = \frac{1}{2}gt_1^2$ or $t_1 = \sqrt{\frac{2d}{g}}$

Upward motion of sound : *There is no effect of gravity in the propagation of sound i.e, always the formula of uniform motion is used for sound or any other wave.*

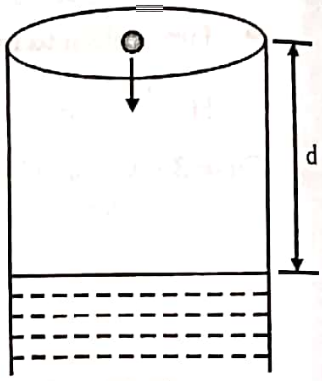


Fig.26

Speed of sound, $v = \frac{d}{t_2}$

(Distance travelled by sound = depth of well = d)

or $t_2 = \frac{d}{v}$

Total time taken to hear the splash, $T = t_1 + t_2 = \sqrt{\frac{2d}{g}} + \frac{d}{v}$

NUMERICAL CHALLENGE 1.8

A person, on the top of a building, throws one stone vertically upwards with a velocity 'u'. He throws an another stone from the same place in the downward direction with a velocity 'u'. Find the ratio of velocities of two stones on the bottom of the building.

Solution

For the stone thrown upward (see fig.24), Initial velocity = + u ; acceleration, a = – g ; distance travelled, s = – H ; final velocity = – v_1.

From second equation of motion, we have, $v^2 = u^2 + 2as$

or $(-v_1)^2 = (+u)^2 + 2(-g)(-H)$ or $v_1^2 = u^2 + 2gH$ or $v_1 = \sqrt{u^2 + 2gH}$ —— (1)

For the stone thrown downward (see fig.24), Initial velocity = – u ; acceleration, a = – g ; distance travelled, s = – H ; final velocity = – v_2.

$v^2 = u^2 + 2as$ or $(-v_2)^2 = (-u)^2 + 2(-g)(-H)$

or $v_2^2 = u^2 + 2gH$ or $v_2 = \sqrt{u^2 + 2gH}$ —— (2)

From eq.(1) and eq.(2), we get that $v_1 = v_2$, therefore $v_1 : v_2 =$ **1 : 1**

0 12.5

NUMERICAL CHALLENGE 1.9

A balloon is ascending at the rate of 5 m/s at a height of 100 m above the ground when a packet is dropped from the balloon. After how much time does it reach the ground ? ($g = 10\ m/s^2$)

Solution

Since the balloon is ascending with velocity 5 m/s, the initial velocity of the packet dropped from the balloon is $u = +5$ m/s ; $a = -g = -10\ m/s^2$; displacement, $s = -100$ m.

From first equation of motion, we have, $s = ut + \frac{1}{2}at^2$ or $-100 = (+5)t + \frac{1}{2}(-10)t^2$

or $-5t^2 + 5t + 100 = 0$ or $t^2 - t - 20 = 0$ or $t^2 - 5t + 4t - 20 = 0$

or $(t - 5)(t + 4) = 0$ or $t = 5$ s and $t = -4$ s (negative time not possible)

Thus, **t = 5 s**. The packet will reach the ground after 5 seconds.

1.8 Graphs in motion

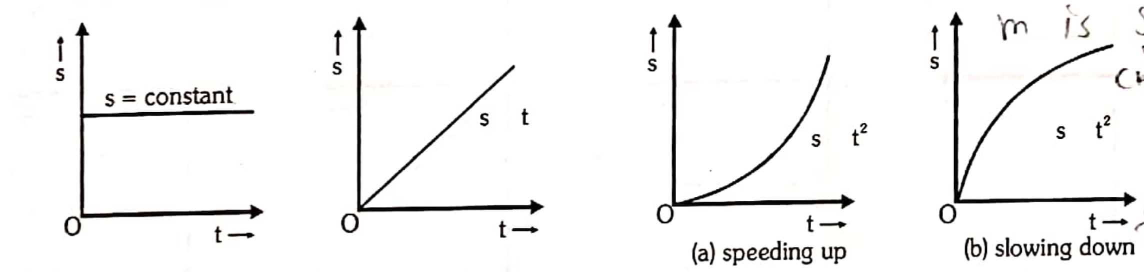
Usually distance-time, position-time, displacement-time, speed-time, velocity-time, acceleration-time graphs are used in understanding motion.

Distance - time graph

Here, distance is taken on y-axis and time is taken on x-axis.

- Distance-time graph is always positive, it is always increasing NEVER decreasing.

$y = mx + c$.
x, y are cordinats
m is slope of line.
c is distance of start of line from origin on y.



A body at rest (s = constant) (v = 0)

A body in uniform motion (s = v × t)

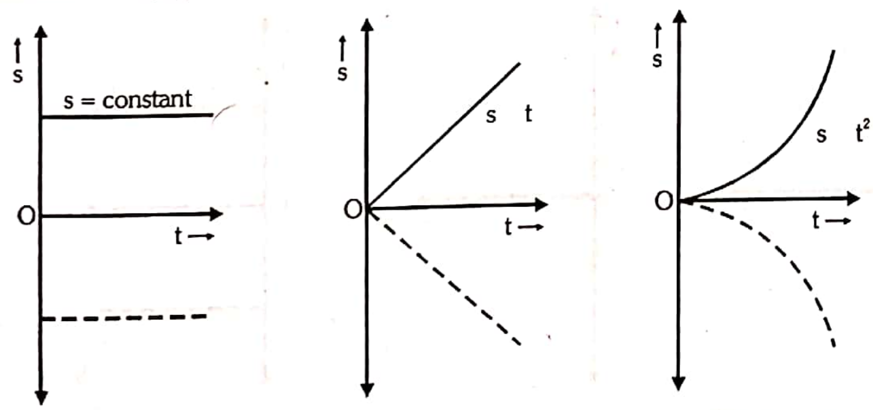
A body in uniformly accelerated motion $(s = ut + \frac{1}{2}at^2)$

Fig.27 Distance-time graphs for different states of motion

Displacement-time graph

Here, displacement is taken on y-axis and time is taken on x-axis.

- Displacement-time graph can be positive or negative, it can be increasing or decreasing.



A body at rest (s = constant) (v = 0)

A body in uniform motion (s = v × t)

A body in uniformly accelerated motion $(s = ut + \frac{1}{2}at^2)$

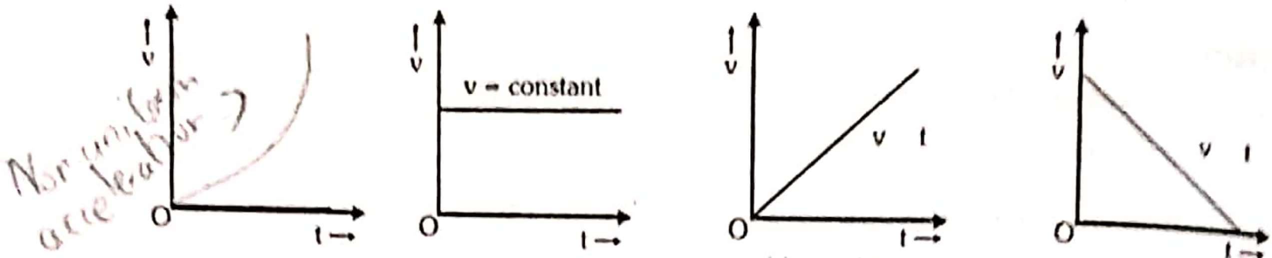
Fig.28 Displacement-time graphs for different states of motion

Speed-time graph

Here, speed is taken on y-axis and time is taken on x-axis.

- Speed-time graph is always positive, it can be increasing or decreasing.

Non uniform acceleration ?



A body at rest (v = 0) | A body in uniform motion (v = constant) | (a) speeding up | (b) slowing down

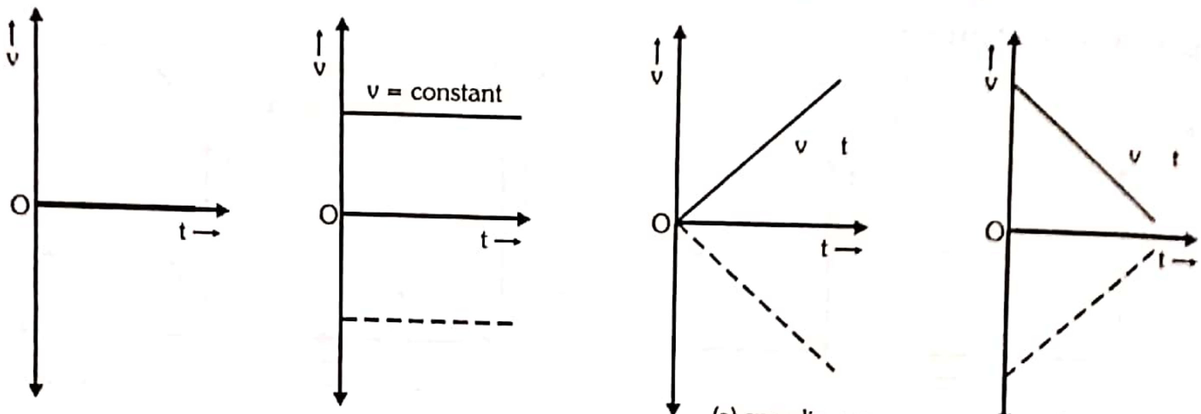
A body in uniformly accelerated motion (v = u + at)

Fig.29 Speed-time graphs for different states of motion

Velocity-time graph

Here, velocity is taken on y-axis and time is taken on x-axis.

- Velocity-time graph can be positive or negative, it can be increasing or decreasing.



(a) speeding up | (b) slowing down

A body at rest (v = 0) | A body in uniform motion (v = constant) | A body in uniformly accelerated motion (v = u + at)

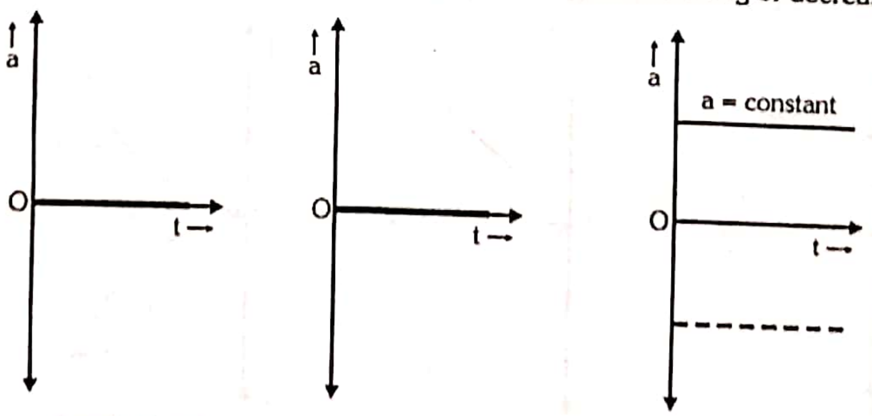
Fig.30 Velocity-time graphs for different states of motion

Acceleration-time graph

area under uniform a = v−u

Here, acceleration is taken on y-axis and time is taken on x-axis.

- Acceleration-time graph can be positive or negative, it can be increasing or decreasing.



A body at rest (a = 0) | A body in uniform motion (a = 0) | A body in uniformly accelerated motion (a = constant)

Fig.31 Acceleration-time graphs for different states of motion

1.9 Significance of graphs in motion

Slope of a graph

Slope of a graph is given by,

$$\text{Slope} = \tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{p}{b}$$

Where, θ is the angle made by the graph with positive x-axis.

- Slope of a graph can be zero, positive, negative or even infinite (∞).
  (1) For $\theta = 0°$, slope is zero (e.g. a horizontal line).
  (2) For $\theta = 90°$, slope is infinite (e.g. a vertical line).
  (3) For $0° < \theta < 90°$, slope is positive (e.g. a line making acute angle with the positive x-axis).
  (4) For $90° < \theta < 180°$, slope is negative (e.g. a line making obtuse angle with the positive x-axis).
  - More the value of θ, more will be the value of $\tan\theta$ i.e., more will be the slope of the graph.

Slope of a straight line graph :

A straight line has a constant slope (see fig.32)

$$\text{Slope} = \tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{p}{b}$$

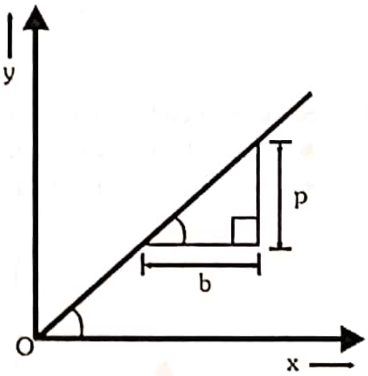


Fig.32 Slope of a straight line graph

Slope of a curved line graph :

The process of finding slopes is more challenging for a curved-line graph because the slope of the curve line changes with the change in the values of variable like x (or time t in motion).

- The slope of a curve line at any point on it is found by making a tangent at that point. If θ be the angle made by that tangent with the positive x-axis, than $\tan\theta$ will be the slope of the curve line at that point (see fig.33).

In the fig.34 (a), the slope of the graph is increasing with the increase in value of x (a concave graph) while in fig.34(b), the slope of the graph is decreasing with the increase in value of x.

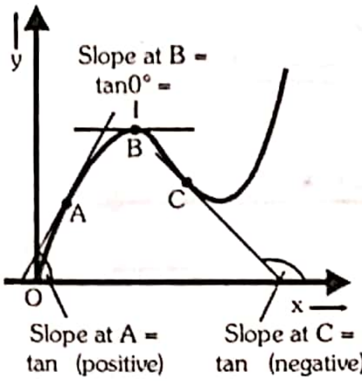


Fig.33 Slope of a curve line graph

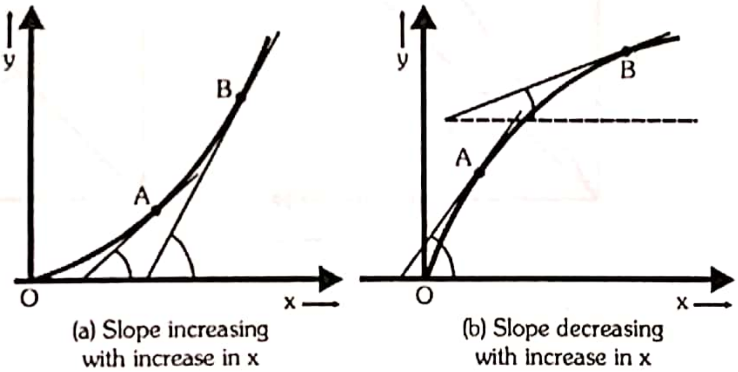


Fig.34 Slope of a curve line graph can be increasing or decreasing

- Slope of distance-time graph gives speed. Slope of displacement-time graph gives velocity.
  - Fig.35 shows a s-t graph in which slope of A is more than slope of B, thus, $v_A > v_B$.
  - From the s-t graph shown in fig.36, we can find the value of v.

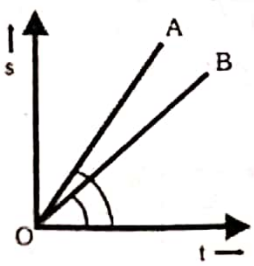


Fig.35

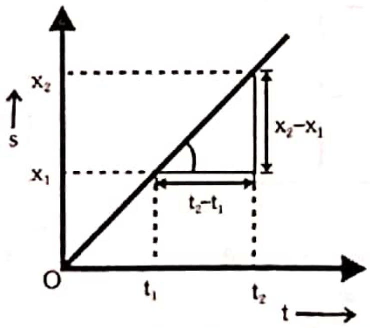
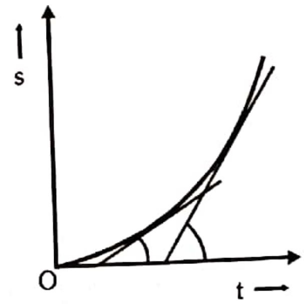


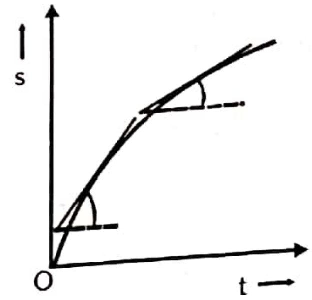
Fig.36

$$\boxed{v = \frac{p}{b} = \frac{x_2 - x_1}{t_2 - t_1}}$$

- In the graphs shown in fig.37, graph 1 represents accelerated motion i.e., v increasing with time. This is because the slope of the graph is increasing with time. Graph 2 represents retarded motion i.e., v decreasing with time. This is because the slope of the graph is decreasing with time.



(a) Graph 1
(v increasing with time)
Accelerated motion



(b) Graph 2
(v decreasing with time)
Retarded motion

Fig.37 Using the concept of slope in s-t graph

- Slope of speed-time graph or velocity-time graph gives acceleration.
  - Fig.38 shows a v-t graph in which, slope of 1 is more than slope of 2, thus, $\mathbf{a_1 > a_2}$.
  - From the v-t graph shown in fig.39, we can find the value of a.

$$\boxed{a = \frac{p}{b} = \frac{v_2 - v_1}{t_2 - t_1}}$$

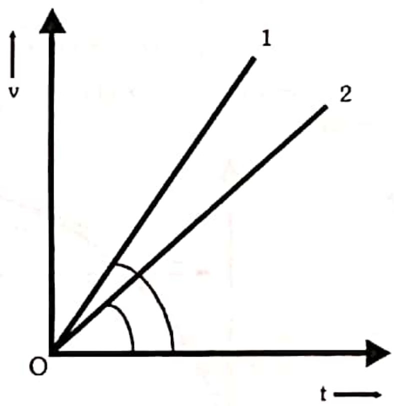


Fig.38

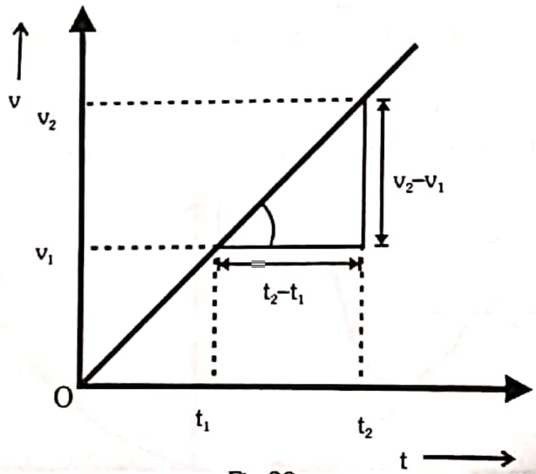
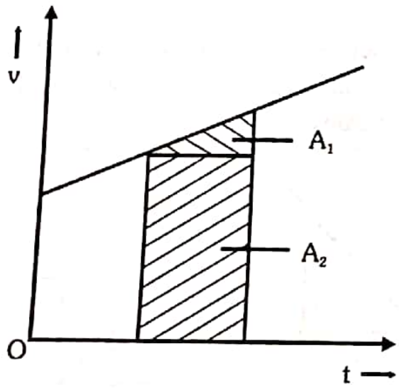


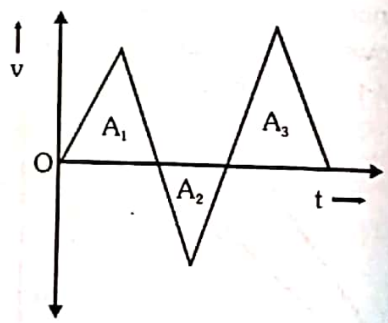
Fig.39

- Total area under the speed-time graph or velocity-time always gives total distance travelled by the body during a given time interval. We can also find displacement using a velocity-time graph [see fig.40(b)].



Distance travelled = $A_1 + A_2$

(a)



Distance travelled = $A_1 + A_2 + A_3$

Displacement = $A_1 - A_2 + A_3$

(b)

Fig.40 Area under v-t graph gives distance travelled by the body.

1.10 Graphs of motion under gravity

We know that upward motion of an object is a retarded motion while downward motion is an accelerated motion. Let us try to make graphs for an object which is thrown upward and returns back to the same height (see fig.41).

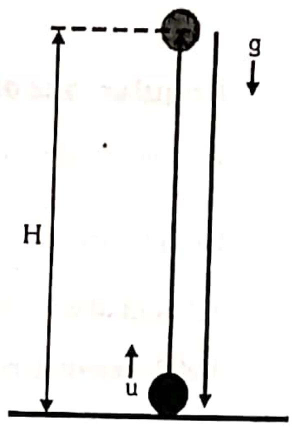
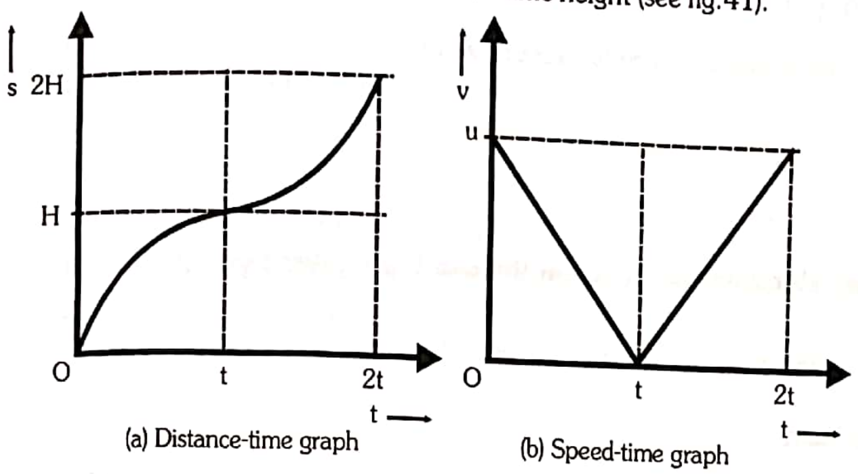
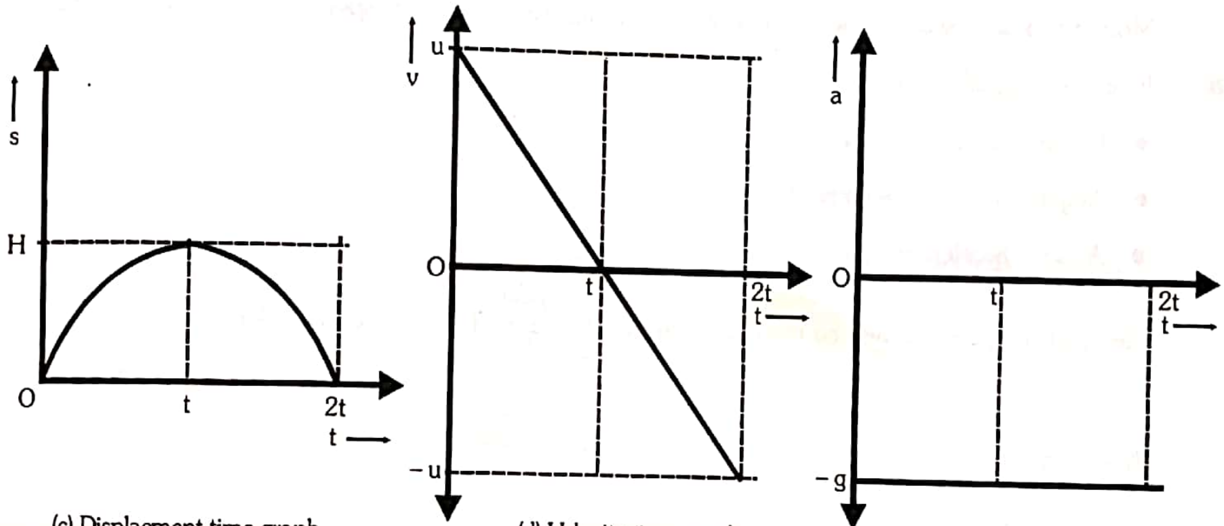


Fig.41





(a) Distance-time graph

(b) Speed-time graph

(c) Displacment-time graph

(d) Velocity-time graph

(e) Acceleration-time graph

Fig.42 Graphs of motion under gravity

- The area under the acceleration-time graph gives change in velocity during a given time interval.

1.11 Circular motion

When a particle moves along a circular path, its motion is called circular motion.

- **A circular motion is always a non-uniform motion** i.e., accelerated motion because the direction of velocity change continuously.
  - Velocity of a particle in circular motion is always tangential to the circular path (see fig.43) i.e., velocity and radius are always ⊥ to each other.

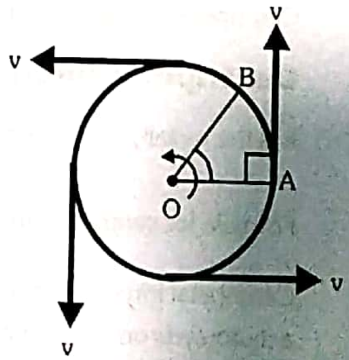


Fig.43 Velocity of a particle along a circular path is tangential to the path

Angular displacement (θ) : The angle described by particle moving along a circular path is called **angular displacement**.

- S.I. unit of angular displacement is **radian**.

π radian = 180°   1 radian = 180°/π = 57.3°

Angular velocity (ω) : The rate of change of angular displacement is called angular velocity.

Formula for ω : $\boxed{\omega = \frac{\theta}{t}}$

S.I. unit of ω : radian per second or rad s^{-1}.

Relation between angular velocity (ω) and linear speed (v) :

$\boxed{v = r\omega}$ (r = radius of circular path)

Angular acceleration (α)

The rate of change of angular velocity is called **angular acceleration**.

Formula for α : $\boxed{\alpha = \frac{\omega_2 - \omega_1}{t}}$

S.I. unit of α : radian/(second)^2 or rad s^{-2}.

Relation between angular acceleration (α) and linear (tangential) acceleration (a_t) :

$\boxed{a = r\alpha}$ (r = radius of circular path)

Uniform circular motion

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.

- In uniform circular motion :
  - Linear speed, v = constant
  - Angular velocity, ω = constant
  - Angular acceleration, α = 0

Here, linear speed can also be found by formula, $\boxed{v = \frac{2\pi r}{T}}$ (T = time period of 1 revolution)

Also, angular velocity ω can be found using formula, $\boxed{\omega = \frac{2\pi}{T}}$

- Uniform circular motion is always an accelerated motion. It has a radially inward acceleration called **centripetal acceleration**.

Formula for centripetal acceleration : $\boxed{a_c = \frac{v^2}{r} = r\omega^2}$

Centripetal acceleration (a_c) and velocity (v) are always perpendicular to each other.

Centripetal force

It is the radially inward force that is required to move an object along a circular path.

Formula for centripetal force : $\boxed{F = ma_c = \frac{mv^2}{r} = mr\omega^2}$

- Centripetal force is always supplied by a real force, the nature of which depends on the situation. While turning a motorcycle on a horizontal circular path, friction provides the necessary centripetal force. The electron moves in a circle around nucleus due to centripetal force provided by the electrostatic force of attraction between positive nucleus and negative electron.
  - While whirling a stone tied with a string, the tension in the string provides the centripetal force. Earth revolves round the Sun due to the centripetal force provided by the gravitational force between the Earth and the Sun.

ALLEN

Non-uniform circular motion

Motion of a particle along the circumference of a circle with a variable speed is called non-uniform circular motion.

- In non-uniform circular motion :
  - Linear speed, $v \neq$ constant
  - Angular velocity, $\omega \neq$ constant
  - Angular acceleration, $\alpha \neq 0$
  - There are two linear accelerations :

    (i) centripetal acceleration (radiailly inward)

    (ii) tangential acceleration (along the tangent or in the direction of velocity)

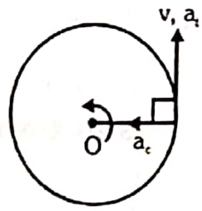


Fig.44 Non-uniform circular motion

Non-uniform circular motion with constant angular acceleration

Equations of motion for the above motion are :

(i) $\omega_2 = \omega_1 + \alpha t$

−1(at)(cb)(de)

(ii) $\theta = \omega_1 t + \dfrac{1}{2}\alpha t^2$

-abcdet

(iii) $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$

(iv) $\theta = \left(\dfrac{\omega_2 + \omega_1}{2}\right)t$

-1(at)-1(cb)-1(de)

-at. -cb. -de

(v) $\omega_{average} = \dfrac{\omega_2 + \omega_1}{2}$

Where, ω_1 = initial angular velocity ; ω_2 = final angular velocity ; θ = distance travelled ; t = time taken.

Angular Displacement in the nth second (i.e., in a particular second) is given by,

$$\boxed{\theta_{nth} = \omega_1 + \frac{1}{2}\alpha(2n-1)}$$

Angular speed, $\omega = 2\pi n$, where, n is number of revolutions per second or the frequency of revolution.

Also, angular speed, $\omega = \dfrac{2\pi}{T}$, where, T is time one revolution (called time period or period).

If a particle is making N revolution per minute (denoted as rpm), angular speed, $\omega = \dfrac{2\pi N}{60}$

NUMERICAL CHALLENGE 1.10

What is the angular velocity in rad/s of the hour, minute and second hand of clock ?

Solution

Time period of revolution of hour hand, T_1 = 12 hours = $12 \times 60 \times 60$ s

Angular velocity of hour hand, $\omega_1 = \dfrac{2\pi}{T_1} = \dfrac{2\pi}{12\times60\times60} = \dfrac{\pi}{21600}$ rad/s

Time period of revolution of minute hand, T_2 = 1 hour = $1 \times 60 \times 60$ s

Angular velocity of hour hand, $\omega_2 = \dfrac{2\pi}{T_2} = \dfrac{2\pi}{1\times60\times60} = \dfrac{\pi}{1800}$ rad/s

Time period of revolution of second hand, T_3 = 1 minute = 1×60 s

Angular velocity of hour hand, $\omega_3 = \dfrac{2\pi}{T_3} = \dfrac{2\pi}{1\times60} = \dfrac{\pi}{30}$ rad/s

NUMERICAL CHALLENGE 1.11

A child pushes a merry-go-round from rest to a final angular speed of 0.50 rev/s with constant angular acceleration. In doing so, the child pushes the merry-go-round 2.0 revolutions. What is the angular acceleration of the merry-go-round ?

Solution

Given, initial angular speed, $\omega_1 = 0$; final no. of revolution/sec or frequency, n = 0.50 rev/s

final angular speed, $\omega_2 = 2\pi n = 2\pi(0.5) = \pi$ rad/s ; angular acceleration, $\alpha = ?$

Total no. of revolution = 2

$\therefore$ Total angle covered, $\theta = 2 \times 2\pi = 4\pi$ rad

Now, using third equation of motion, $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, we get,

$(\pi)^2 = (0)^2 + 2\alpha(4\pi)$

or $\pi^2 = 8\alpha\pi$

or $\alpha = \pi/8$ **rad/s²**

EXERCISE

Multiple choice questions

1. A body goes from A to B with a velocity of 20 m/s and comes back from B to A with a velocity of 30 m/s. The average velocity of the body during the whole journey is

(1) zero (2) 25 m/s (3) 24 m/s (4) none of these

2. A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 min to reach the field, what is the average velocity of farmer during the walk?

(1) 35 m/min. (2) 45 m/min. (3) 25 m/min. (4) 55 m/min.

3. A rubber ball dropped from a certain height is an example of

(1) non-uniform acceleration (2) uniform retardation

(3) uniform speed (4) non-uniform speed

4. For motion on a straight line path with constant acceleration, the ratio of the magnitude of the displacement to the distance covered is

(1) $= 1$ (2) ≥ 1 (3) ≤ 1 (4) < 1

5. An object is moving with velocity 10 m/s. A constant force acts for 4 s on the object and gives it a speed of 2 m/s in opposite direction. The acceleration produced is

(1) 3 m/s^2 (2) –3 m/s^2 (3) 6 m/s^2 (4) –6 m/s^2

6. A car is moving along a circular path with a uniform speed 54 kmph. Find the difference in the velocities of the car when it is at the diametrically opposite points.

(1) 54 kmph (2) 108 kmph (3) 81 kmph (4) 27 kmph

7. A point traversed $3/4^{th}$ of the circle of radius R in time t. The magnitude of the average velocity of the particle in this time interval is

(1) $\frac{\pi R}{t}$ (2) $\frac{3\pi R}{2t}$ (3) $\frac{R\sqrt{2}}{t}$ (4) $\frac{R}{\sqrt{2}t}$

8. Which of the following statements is false?

(1) A body can have zero velocity and still be accelerated.

(2) A body can have a constant velocity and still have a varying speed.

(3) A body can have a constant speed and still have a varying velocity.

(4) The direction of the velocity of a body can change when its acceleration is constant.

9. A particle moves along the side AB, BC, CD of a square of side 25 m with a velocity of 15 m/s. Its average velocity is

(1) 15 ms^{-1} (2) 10 ms^{-1} (3) 7.5 ms^{-1} (4) 5 ms^{-1}

10. If an object covers distances directly proportional to the square of the time lapsed, then the acceleration is

(1) increasing (2) decreasing (3) constant (4) none of these

11. A stone weighing 3 kg falls from the top of a tower 100 m high and buries itself 2 m deep in the sand. The time of penetration is

(1) 0.09 sec (2) 0.9 sec (3) 2.1 sec (4) 1.3 sec

12. The velocity of a body at any instant is 10 m/s. After 5 sec, velocity of the particle is 20 m/s. The velocity at 3 seconds before that instant is

(1) 8 m/sec (2) 4 m/sec (3) 6 m/sec (4) 7 m/sec

13. A body covers 200 cm in the first 2 sec and 220 cm in next 4 sec. What is the velocity of the body at the end of 7^{th} second?

(1) 40 cm/sec (2) 20 cm/sec (3) 10 cm/sec (4) 5 cm/sec

14. A body falls from a height h = 200 m. The ratio of distance travelled in each 2 s, during t = 0 to t = 6 s of the journey is

(1) 1 : 4 : 9 (2) 1 : 2 : 4 (3) 1 : 3 : 5 (4) 1 : 2 : 3

15. A stone is thrown vertically upward with an initial velocity u from the top of a tower. It reaches the gro with a velocity 3u. The height of the tower is

(1) $\frac{3u^2}{g}$ (2) $\frac{4u^2}{g}$ (3) $\frac{6u^2}{g}$ (4) $\frac{9u^2}{g}$

16. A particle is moving in a straight line with initial velocity u and uniform acceleration a. If the sum of the distan travelled in t^{th} and $(t + 1)^{th}$ seconds is 100 cm, then its velocity after t seconds in cm/s is

(1) 20 (2) 30 (3) 50 (4) 80

17. A body freely falling from rest has velocity v after it falls through a height h. The distance it has to fall dow further for its velocity to become double is

(1) 4h (2) 6h (3) 3h (4) 10h

18. A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its moti is (g = 10 m/s²)

(1) 25 m (2) 45 m (3) 90 m (4) 125 m

19. A stone is dropped from the top of a tower. If it travels 34.3 m in the last second before it reaches the groun find the height of the tower. (g=9.8 m/s²)

(1) 39.2 m (2) 58.8 m (3) 78.4 m (4) 98 m

20. A freely falling object falls through a height h in the n^{th} second. What is the fall of height in the next second?

(1) h − g (2) hg (3) h + g (4) $\frac{h}{g}$

21. A stone is dropped from a certain height and another stone is dropped from the same height after 2 s. Wh will be their separation after 10 more seconds?

(1) 115.6 m (2) 156.5 m (3) 172.3 m (4) 215.6 m

22. A body falls from a height of 100 m. After 2 seconds if gravity disappears, find the total time it would tak to reach the ground (take g = 10 ms⁻²).

(1) 2 s (2) 4 s (3) 6 s (4) 8 s

23. A body is falling freely. If the displacement in the last second is equal to the displacement in the first 3 second find the time of free fall.

(1) 5 s (2) 10 s (3) 15 s (4) 20 s

24. An object is thrown vertically up with a velocity of 49 ms⁻¹. How high will it rise?

(1) 98 m (2) 117.6 m (3) 122.5 m (4) 137.2 m

25. A body thrown vertically upward remains in air for 2 seconds. Another body is thrown vertically upward wi double the velocity. How long does it stay in air?

(1) 4 s (2) 8 s (3) 16 s (4) 32 s

26. A stone is thrown vertically up with an initial velocity 49 ms⁻¹ from the top of a tower and reaches ground aft 12 seconds. Find the height of the tower.

(1) 98 m (2) 117.6 m (3) 137.2 m (4) 156.8 m

27. Two stones are projected from the top of a tower 100 m high each with a veloci 10 ms⁻¹. One is projected vertically up and the other vertically down. Find the ratio of the speeds with whi they strike the ground.

(1) 1 : 10 (2) 10 : 1 (3) 1 :1 (4) 2 :1

28. A ball is projected vertically up from the foot of a tower of height 100 m with a velocity of 40 ms⁻¹. At the same instant another ball is dropped from the top of the tower. When and where do they meet each other? (take g = 10 ms⁻²)

(1) 2.5 s ; 68.75 m from ground (2) 2 s ; 65 m from ground
(3) 3 s ; 75 m from ground (4) 3.5 s ; 85 m from ground

29. An object is projected vertically up from the top of a tower of height 58.8 m with an initial velocity 4.9 ms⁻¹. Calculate the time of flight of the object.

(1) 2 s (2) 4 s (3) 6 s (4) 8 s

30. If the time of fall of two objects are in the ratio 1 : 2, find the ratio of the heights from which they fall.

(1) 1: 2 (2) 2: 1 (3) 1: 4 (4) 4: 1

31. An object is dropped from a balloon rising up with a velocity 2 ms⁻¹. Find the velocity of the object after 2 seconds of its release. (take g = 10 ms⁻²)

(1) 9 ms⁻¹ (2) 18 ms⁻¹ (3) 27 ms⁻¹ (4) 36 ms⁻¹

32. A ball is dropped from the top of a building. The ball takes 0.5 sec to fall past the 3 m height of a window some distance from the top of the building. If the speed of the ball at the top and the bottom of the window are v_T and v_B respectively, then (g = 9.8 m/s²)

$S_n = 5(2n-1)$

(1) $v_T + v_B = 12$ m/s (2) $v_T - v_B = 4.9$ m/s (3) $v_T v_B = 1$ m/s (4) $\frac{v_B}{v_T} = 1$ m/s

33. An object dropped from the top of a tower covers in the last second, seven times the distance it covered in the first second. Find the time of flight.

(1) 2 s (2) 3 s (3) 4 s (4) 5 s

34. A stone is dropped into water from a bridge of height 44.1 m above the water level. Another stone is thrown into water 1 second later. If both strike the water simultaneously, find the initial speed of the second stone.

(1) 12.25 ms⁻¹ (2) 12.5 ms⁻¹ (3) 12.75 ms⁻¹ (4) 13 ms⁻¹

35. A stone is projected up with a velocity 'u' and at the same time another is dropped from a height $2u$. When will they meet in air?

(1) 1 s (2) 2 s (3) 3 s (4) 4 s

36. Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 s the distance between them is

(1) 4.9 m (2) 19.6 m
(3) 9.8 m (4) 39.2 m

37. A train running at a speed of 120 kmph is approaching a station. Driver applies brakes just 200 m before the station to stop it at the station. Find the retardation of the train.

(1) $\frac{25}{9}$ ms⁻² (2) $\frac{30}{11}$ ms⁻² (3) $\frac{37}{13}$ ms⁻² (4) $\frac{41}{11}$ ms⁻²

38. A bullet fired into a fixed wooden target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest, if it experiences a constant deceleration?

(1) 1 cm (2) 2 cm (3) 3 cm (4) 4 cm

39. A particle under the action of a constant force moves from rest upto 20 seconds. If distance covered in first 10 seconds is s_1 and that covered in next 10 seconds is s_2 then

(1) $s_1 = s_2$ (2) $s_2 = 3s_1$ (3) $s_2 = 2s_1$ (4) $s_2 = 4s_1$

40. A ball is thrown vertically upward. It has a speed of 10 m/sec when it has reached one half of its maximum height. How high does the ball rise? (Take g = 10 m/s²)

(1) 10 m (2) 5 m (3) 15 m (4) 20 m

41. A body moves with a uniform acceleration a and zero initial velocity. Another body B starts from the same point and moves in the same direction with a constant velocity v. The two bodies meet after a time 't'. The value of t is

(1) $\frac{2v}{a}$ (2) $\frac{v}{a}$ (3) $\frac{v}{2a}$ (4) $\sqrt{\frac{v}{2a}}$

42. Two balls are dropped from height h and 2h respectively. The ratio of times of these balls to reach the earth is

(1) $1:\sqrt{2}$ (2) $\sqrt{2}:1$ (3) 2:1 (4) 1:4

43. A car moving with a speed of 50 km/hour can be stopped by brakes after a distance 6 m. If the same car is moving at a speed of 100 km/hour, the minimum stopping distance is

(1) 6 m (2) 12 m (3) 18 m (4) 24 m

44. A ball is dropped from the roof of a tower of height h. The total distance covered by it in the last second of its motion is equal to the distance covered by it in first three seconds. The value of h in meters is ($g = 10\ m/s^2$)

(1) 125 (2) 200 (3) 100 (4) 80

45. A balloon is flying up with a constant velocity of 5 m/s. At a height of 100 m, a stone is dropped from it. At the instant the stone reaches the ground level, the height of the balloon will be

(1) 25 m (2) 0 m (3) 125 m (4) 100 m

46. A stone is thrown vertically up from the ground. It reaches a maximum height of 50 meters in 10 sec. After what time it will reach the ground?

(1) 10 sec (2) 20 sec (3) 30 sec (4) 40 sec

47. A particle starts sliding down a frictionless inclined plane. If s_n is the distance travelled by it from time $t = (n-1)$ sec to $t = n$ sec, the ratio $\frac{s_n}{s_{n+1}}$ is

(1) $\frac{2n-1}{2n+1}$ (2) $\frac{2n+1}{2n}$ (3) $\frac{2n}{2n+1}$ (4) $\frac{2n+1}{2n-1}$

48. If a body starts from rest and travels 120 m in the 8^{th} second, then its acceleration is

(1) $16\ m/s^2$ (2) $10\ m/s^2$ (3) $0.227\ m/s^2$ (4) $0.03\ m/s^2$

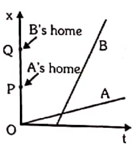
49. A particle travels 10 m in first 5 s and 10 m in next 3 s. Assuming constant acceleration, what is the distance travelled in next 2 s?

(1) 8.3 m (2) 9.3 m (3) 10.3 m (4) None of these

50. Initially a body is at rest. If its acceleration is $5\ ms^{-2}$ then the distance travelled in the 18^{th} second is

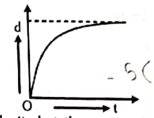
(1) 86.6 m (2) 87.5 m (3) 88 m (4) 89 m

51. Figure shows the position-time (x-t) graph of the motion of two boys A and B returning from their school O to their homes P and Q respectively. Which of the following statements is true?



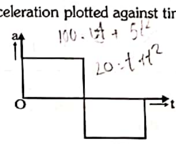
(1) A walks faster than B
(2) Both A and B reach home at the same time
(3) B starts for home earlier than A
(4) B overtakes A on his way to home

52. The distance of a particle as a function of time is shown below. The graph indicates that

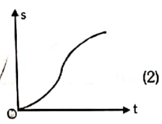
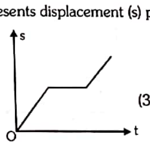
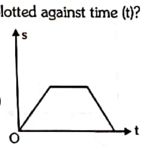
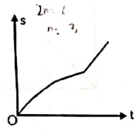


(1) The particle starts with certain velocity but the motion is retarded and finally the particle stops
(2) The velocity of the particle is constant throughout
(3) The acceleration of the particle is constant throughout in the direction of motion
(4) The particle starts with some constant velocity, the motion is accelerated, and finally the particle moves with some constant velocity.

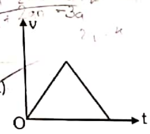
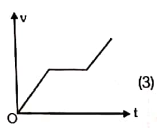
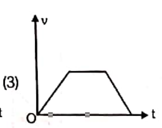
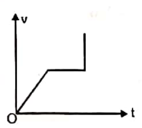
53. A particle starts from rest and its acceleration plotted against time (t) is shown below.



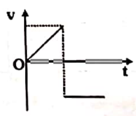
Which of the following represents displacement (s) plotted against time (t)?

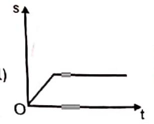
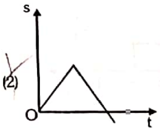
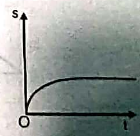
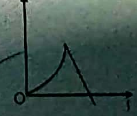
(1) 
(2) 
(3) 
(4) 

54. In question 53, which of the following will represent velocity (v) plotted against time (t)?

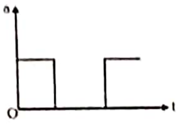
(1) 
(2) 
(3) 
(4) 

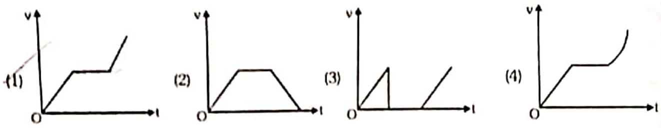
55. The velocity-time graph for a particle moving along x-axis is shown in the figure. The corresponding displacement -time graph is correctly shown by



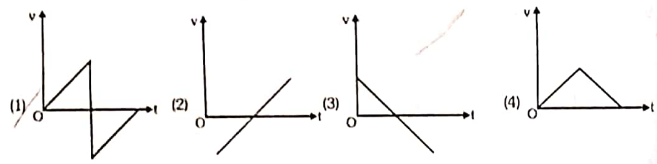
(1) 
(2) 
(3) 
(4) 

56. Which of the following graphs would probably show the velocity plotted against time graph for a body whose acceleration-time graph is shown in the figure?

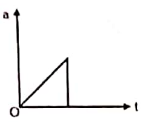


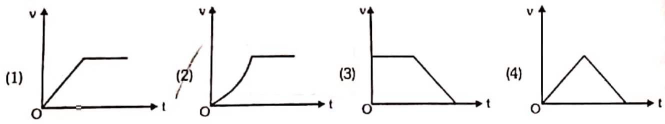


57. The velocity-time graph of a body falling from rest under gravity and rebounding from a solid surface is represented by which of the following graphs?

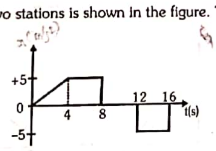


58. The acceleration-time graph for a body is shown in the figure. The most probable velocity-time graph for the body is



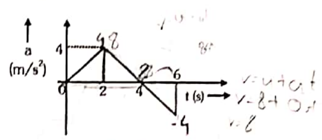


59. The acceleration of a train between two stations is shown in the figure. The maximum speed of the train is



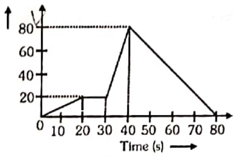
(1) 60 m/s (2) 30 m/s (3) 120 m/s (4) 90 m/s

60. Acceleration-time graph for a particle moving in a straight line is as shown in figure. Change in velocity of the particle from t = 0 to t = 6 s is



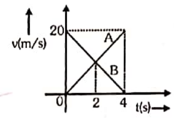
(1) 10 m/s (2) 4 m/s (3) 12 m/s (4) 8 m/s

61. The v-t graph of a moving object is given in figure. The maximum acceleration is



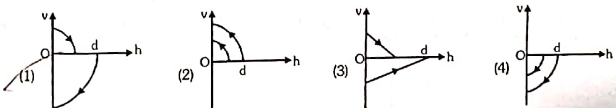
(1) 1 cm/s² (2) 2 cm/s² (3) 3 cm/s² (4) 6 cm/s²

62. Speed-time graph of two cars A and B approaching towards each other is shown in figure. Initial distance between them is 60 m. The two cars will cross each other after time



(1) 2 s (2) 3 s (3) 1.5 s (4) $\sqrt{2}$ s

63. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $\frac{d}{2}$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as



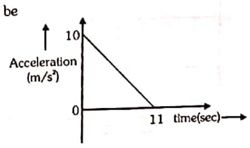
64. A body starts from rest at time t = 0. The acceleration – time graph is shown in the figure. The maximum velocity attained by the body will be

(1) 110 m/s

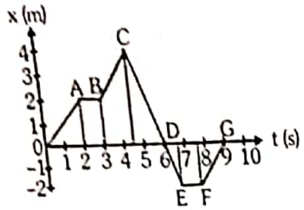
(2) 55 m/s

(3) 650 m/s

(4) 550 m/s

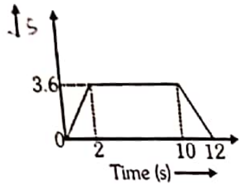


A dancer is demonstrating dance steps along a straight line. The position-time graph is given below. The average velocity of the dancer during time interval between t = 4.5 s to t = 9 s is



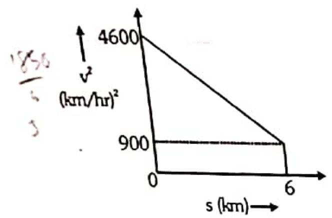
(1) 1 ms^{-1} (2) – 1.33 ms^{-1} (3) 2.75 ms^{-1} (4) –0.89 ms^{-1}

66. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



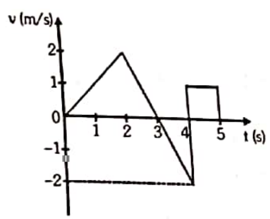
(1) 3.6 m (2) 28.8 m
(3) 36.0 m (4) Cannot be calculated from the above graph

67. A graph between the square of the velocity of a particle and the distance (s) moved is shown in figure. The acceleration of the particle in kilometres per hour square is



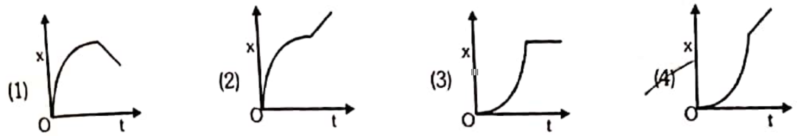
(1) 225 (2) 308.3 (3) – 225 (4) – 308.3

68. The velocity versus time graph of a body moving along a straight line is as shown in fig. The ratio of displacement and distance covered by body in 5 seconds is

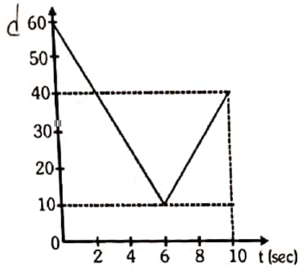


(1) 2 : 3 (2) 3 : 5 (3) 1 : 1 (4) 1.5 : 5

69. A car starts from rest, accelerates uniformly for 4 seconds and then moves with uniform velocity. Which of the x-t graphs represents the motion of the car?



70. The fig. shows the displacement-time graph of a particle moving on a straight line path. What is the magnitude of average velocity of the particle over 10 seconds?



(1) 2 ms^{-1} (2) 4 ms^{-1} (3) 6 ms^{-1} (4) 8 ms^{-1}

71. The earth's radius is 6400 km. It makes one rotation about its own axis in 24 hrs. The centripetal acceleration of a point on its equator is nearly

(1) 340 cm/s^2 (2) 34 cm/s^2 (3) 3.4 cm/s^2 (4) 0.34 cm/s^2

72. The acceleration of a point on the rim of flywheel 1 m in diameter, if it makes 1200 revolutions per minute, is

(1) $8\pi^2$ m/s^2 (2) 80 π^2 m/s^2 (3) 800 π^2 m/s^2 (4) none of these

73. A phonograph record on turn table rotates at 30 rpm. The linear speed of a point on the record at the needle at the beginning of the recording when it is at a distance of 14 cm from the centre is

(1) 22 cm/sec (2) 44 cm/sec (3) 48 cm/sec (4) 52 cm/sec

74. A particle is acted upon by a constant force, the direction of which is always perpendicular to the velocity of particle. The motion of particle takes place in same plane. From the above statement it implies

(1) Particle is moving in a circular path
(2) Magnitude of its acceleration is constant
(3) Its velocity is uniform
(4) Both (1) and (2)

75. A body moves along the circumference of a circular track. It returns back to its starting point after completing the circular track twice. If the radius of the track is R, the ratio of displacement to the distance covered by the body will be

(1) 0 (2) $8\pi R$ (3) $\sqrt{3R}$ (4) $\frac{\pi}{R}$

76. Two cars are going round curves, one car travelling at 60 km/hr and the other at 30 km/hr. Each car experiences the same centripetal acceleration. The radii of the two curves are in the ratio

(1) 4 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

77. A fan is making 600 revolutions/minute. If it makes 1200 revolutions/minute, what is the increase in its angular velocity?

(1) 10π rad/sec (2) 20π rad/sec (3) 60π rad/sec (4) 40π rad/sec

78. A stone tied to the end of a 20 cm long string is whirled in a horizontal circle. If the centripetal acceleration is 9.8 m/s², its angular speed in rad/sec is

m 0.2 $9.8 = 0.2 . \omega^2$

(1) $\frac{22}{7}$ (2) 7 (3) 14 (4) 20

79. The ratio of angular speed of minute's hand and hour's hand of a watch is

(1) 1 : 6 (2) 6 : 1 (3) 1 : 12 (4) 12 : 1

80. A point on the rim of a wheel 3 m in diameter has linear velocity of 18 m/sec. The angular velocity of the wheel is given by

(1) 12 rad/s (2) 10 rad/s (3) 8 rad/s (4) 6 rad/s

81. A particle is moving along a circular path of radius 5 m with a uniform speed 5 ms^{-1}. What will be the average acceleration when the particle completes half revolution?

(1) zero (2) 10 ms^{-2} (3) 10π ms^{-2} (4) $\frac{10}{\pi}$ ms^{-2}

82. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same length of time t. The ratio of angular speed of the first car to that of the second car is

(1) $m_1 : m_2$ (2) $r_1 : r_2$ (3) 1 : 1 (4) $m_1r_1 : m_2r_2$

83. The angular velocity of a wheel is 70 rad/s. If the radius of the wheel is 0.5 m, then linear velocity of the wheel is

(1) 70 m/s (2) 35 m/s (3) 30 m/s (4) 20 m/s

84. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. What is the linear speed of the motion?

(1) 2.3 cm/s (2) 5.3 cm/s (3) 0.44 cm/s (4) None of these

85. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 22 s, then the acceleration of the stone is

(1) 5 m/s² (2) 10 m/s² (3) 12.8 m/s² (4) None of these

ANSWERS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	[illegible]
Ans.	1	3	4	1	2	2	3	2	4	3	1	2	3	3	2	3	3	2	3	[illegible]
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	[illegible]
Ans.	4	3	1	3	1	2	3	1	2	3	2	1	3	1	2	3	1	1	2	[illegible]
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	[illegible]
Ans.	1	1	4	1	3	1	1	1	1	2	4	1	1	1	4	1	1	2	2	[illegible]
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	[illegible]
Ans.	4	2	1	2	4	3	4	2	4	1	3	3	2	4	1	1	2	2	4	[illegible]
Que.	81	82	83	84	85															
Ans.	4	3	2	2	3															